PROBLEM 1 :

Let a,b,c,d be positive réal numbers.

Prove the inequality

$$a^4+b^4+c^4+d^4-4abcd \geq 4(a-b)^2\sqrt{abcd}$$

PROBLEM 2 :

Let a, b, c be positive real numbers such that a + b + c = 1.

Prove the inequality

$$(a+b)2(1+2c)(2a+3c)(2b+3c) \geq 54abc.$$

PROBLEM 3:

Let a, b, c be positive real numbers such that a + b + c = 6.

Prove the inequality

$$\sqrt[3]{ab+bc}+\sqrt[3]{bc+ca}+\sqrt[3]{ca+ab} \leq 6.$$